

Basic cosmology

Dedicated to Halton C. Arp

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Abstract

Basic cosmology describes the universe as a Robertson-Walker model filled with black-body radiation and no barionic matter, and as observational data it uses only the value of the speed of light, the Hubble and deceleration parameters and the black-body temperature at the present epoch. It predicts the value of the next new parameter in the Hubble law.

The Robertson-Walker model.

Its line-element is:

$$ds^2 = -dt^2 + \frac{1}{c^2} \left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) \quad (1)$$

where $c \equiv c(t)$ with $c_0 = 299792458.0$ m/s is the speed of light at the present epoch, $(t = 0)$ ¹.

$T \equiv T(t)$, with $T_0 = 2.74K$ being the temperature of the black-body radiation, the mass density $\rho \equiv \rho(t)$ and pressure $P \equiv P(t)$ are:

$$\rho = \frac{a}{c^2} T^4, \quad P = \frac{1}{3} \rho c^2 \quad (2)$$

where $a = 7.565767 \cdot 10^{-16}$ J/m³/K⁴ is the radiation constant. No other mass contributes to the dynamics of the model. I assume that either it can be neglected because of its particular fractal distribution or because its content being highly unreliable now it is advisable to postpone its consideration to a non basic model.

Under the preceding assumptions Einstein's equations:

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} - \Lambda g_{\alpha\beta} = -\frac{8\pi G}{c^2} (\rho c^2 u_\alpha u_\beta + P(g_{\alpha\beta} + u_\alpha u_\beta)) \quad (3)$$

where Λ is the cosmological constant with dimensions T⁻², reduce to the following two equations:

$$Eq_1 \equiv 3\dot{c}^2 + 3kc^4 - \Lambda c^2 = 8\pi G \rho c^2 \quad (4)$$

and:

$$Eq_2 \equiv 2c\ddot{c} - 5\dot{c}^2 - kc^4 + \Lambda c^2 = 8\pi G P \quad (5)$$

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¹ $f(t)$ being a function of t , f_0 means $f(0)$

Using (2) the following linear combination $1/2(Eq_1 - 3Eq_2) = 0$ becomes:

$$Eq_3 \equiv c\ddot{c} - 3\dot{c}^2 - kc^4 + \frac{2}{3}\Lambda c^2 = 0 \quad (6)$$

an equation that will be a useful substitute of Eq_2 once Λ and k will be known.

Solving the system of algebraic equations (4) and (5) with unknowns Λ and k we get:

$$\Lambda = \frac{4\pi G}{c^2}(\rho c^2 + 3P) - \frac{3}{c^2}(c\ddot{c} - 2\dot{c}^2) \quad (7)$$

and:

$$k = \frac{4\pi G}{c^4}(\rho c^2 + P) - \frac{3}{c^4}(c\ddot{c} - \dot{c}^2) \quad (8)$$

Hubble's law.

Light emitted at time t_e at some point with coordinate r_e propagating in the radial direction and being received at time t_r at a point with radial coordinate r_r travels a proper distance L :

$$L = \int_{r_e}^{r_r} \frac{dr}{\sqrt{1 - kr^2}}, \quad \text{or} \quad L = - \int_{t_r}^{t_e} c(t) dt, \quad (9)$$

If δt_e is the period of the light emitted at r_e and δt_r is the period of the light received at r_r then from the preceding formula it follows that:

$$0 = c_r \delta t_r - c_e \delta t_e \quad (10)$$

from where it follows that defining the red shift z by:

$$z = \frac{\delta t_r}{\delta t_e} - 1 \quad (11)$$

we get:

$$z = \frac{c_e}{c_r} - 1 \quad (12)$$

The integral defining L can be approximated as²:

$$L = -c_r(t_e - t_r) - \frac{1}{2}\dot{c}_r(t_e - t_r)^2 - \frac{1}{6}\ddot{c}_r(t_e - t_r)^3 \quad (13)$$

and similarly c_e can be approximated by:

$$c_e = c_r + \dot{c}_r(t_e - t_r) + \frac{1}{2}\ddot{c}_r(t_e - t_r)^2 + \frac{1}{6}\ddot{c}_r(t_e - t_r)^3, \quad (14)$$

Inverting (13) we get:

$$t_e - t_r = -\frac{1}{c}L - \frac{1}{2}\frac{\dot{c}}{c^3}L^2 + \frac{1}{6}\frac{c\ddot{c} - 3\dot{c}^2}{c^5}L^3 \quad (15)$$

and substitution of (15) in (14) and the result in (12) we get:

²Dots overhead mean derivatives with respect to t .

$$z = -\frac{\dot{c}}{c^2}L + \frac{1}{2}\frac{c\ddot{c} - \dot{c}^2}{c^4}L^2 + \frac{1}{6}\left(\frac{4c\ddot{c}\dot{c} - 3\dot{c}^3 - c^2\ddot{\dot{c}}}{c^6}\right)L^3 \quad (16)$$

Defining the Hubble function H and the deceleration parameter q as usual, and the jerk parameter j ³

$$H = -\frac{\dot{c}}{c}, \quad q = \frac{c\ddot{c}}{\dot{c}^2} - 2, \quad j = 6 - 6\frac{c\ddot{c}}{\dot{c}^2} + \frac{c^2\ddot{\dot{c}}}{\dot{c}^3} \quad (17)$$

we extend with an extra term the well known Hubble formula:

$$z = \frac{H}{c}L + \frac{1}{2}\frac{H^2}{c^2}(1+q)L^2 + \frac{1}{6}\frac{H^3}{c^3}(6+6q+j)L^3 \quad (18)$$

The formulas (7)) and (8) can be written:

$$\Lambda = \bar{\Lambda} + 4\pi G \left(\rho + \frac{3P}{c^2} \right), \quad \text{where } \bar{\Lambda} = -3H^2q \quad (19)$$

and:

$$k = \bar{k} + \frac{4\pi G}{c^2} \left(\rho + \frac{P}{c^2} \right) \quad \text{where } \bar{k} = -\frac{H^2(1+q)}{c^2} \quad (20)$$

Observational data

The Hubble constant and the deceleration parameter have been measured to be $H_0 = 72$ km/s/Mpc and $q_0 = -0.55$. This is all that is needed with c_0 and T_0 to derive the values of the cosmological constant Λ and the curvature constant k using Eqs. (19) and (20).

For $t = 0$ the r-h-s of these two equations are known:

$$\bar{\Lambda}_0 = 8.983543533 \cdot 10^{-36} \text{ s}^{-2}, \quad \bar{k}_0 = -2.726056423 \cdot 10^{-53} \text{ m}^{-2} \quad (21)$$

and:

$$\rho_0 = 4.473697482 \cdot 10^{-31} \text{ kg m}^{-3}, \quad P_0 = 1.340252927 \cdot 10^{-14} \text{ kg m}^{-1} \text{ s}^{-2} \quad (22)$$

and therefore the constants Λ and k are:

$$\Lambda = 8.984293774 \cdot 10^{-36} \text{ s}^{-2}, \quad k = -2.725499919 \cdot 10^{-53} \text{ m}^{-2} \quad (23)$$

With Λ and k known, from (6) we get:

$$\ddot{c} = \frac{1}{3} \left(\frac{3c^4k + 9\dot{c}^2 - 2c^2\Lambda}{c} \right) \quad (24)$$

Differentiating now with respect to t we have:

$$\dot{\ddot{c}} = \dot{c} \left(9c^2k + \frac{15\dot{c}^2}{c^2} - \frac{14}{3}\Lambda \right) \quad (25)$$

³ $j = a^2\ddot{\dot{a}}/\dot{a}^3$ if $a = c_0/c$ is the scale factor.

and using now the definitions (17) we find the following convenient expression for the new function:

$$j = \frac{1}{3} \left(\frac{9H^2 + 9c^2k - 2\Lambda}{H^2} \right) \quad (26)$$

Therefore the predicted observational value of $j(0)$ is⁴:

$$j(0) = 0.55 \quad (27)$$

*Maximally symmetric models*⁵

Let us assume that at some value of t we know the values of c , \dot{c} and \ddot{c} or equivalently, from (17), the values of c , H and q corresponding to some general function $c(t)$. These data are sufficient to calculate the Riemann, and Einstein's tensors of the the line-element (1) at this time t . I call Osculating model, [3], at time t the Robertson-Walker model with line-element:

$$ds^2 = -dt^2 + \frac{1}{\bar{c}^2} \left(\frac{dr^2}{1 - \bar{k}r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right) \quad (28)$$

solution of Einstein's equations:

$$R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} - \bar{\Lambda}g_{\alpha\beta} = 0 \quad (29)$$

where $\bar{\Lambda}$ and \bar{k} are defined in (19) and (20). This is a vacuum solution but it is more than that: it is one of the space-times with maximum symmetry. This meaning that the Riemann tensor is:

$$R_{\alpha\beta\mu\nu} = -\frac{1}{3}\bar{\Lambda}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}) \quad (30)$$

and therefore it is invariant under a 10 dimensional group of isometries.

The concept of osculating model allow us to look at the history of the Universe as a continuous unfolding of maximally symmetric space-times while we live at an epoch when our osculating universe is one of de Sitter's space-time models, type dS_- , with:

$$\bar{c} = \frac{\bar{\lambda}_0}{\bar{p}_0} \operatorname{csch} \left(\bar{\lambda}_0 t + \operatorname{csch}^{-1} \left(\frac{\bar{p}_0 c_0}{\bar{\lambda}_0} \right) \right), \quad \bar{\lambda}_0 = \sqrt{\frac{\bar{\Lambda}_0}{3}}, \quad \bar{p}_0 = \sqrt{-\bar{k}_0} \quad (31)$$

and the values of Λ and k are so close to the values of $\bar{\Lambda}_0$ and \bar{k}_0 that the functions c and \bar{c} are almost undistinguishable in the interval $t = -1..0$. The Figures 1-4 are in succession the graphs of c , ρ , $\bar{\Lambda}$ and \bar{k} . The units are such that: $c_0 = 1$, $H_0 = 1$, $\rho_0 = 1.4 \cdot 10^{-4}$, $\Lambda_0 = 1.65$, $k_0 = -0.45$ and $8\pi G = 1$.

Hamiltonian formalism

The differential equation (6) describes the ensemble $\mathcal{E}(\Lambda, k)$ of Robertson-Walker models (1) when its source is such that:

⁴this value is quite compatible with one of the determinations of $j_0 = 0.631 \pm 0.290$ mentioned in **Table 3** of reference [5]

⁵See [1] for instance

$$P = \frac{1}{3}\rho c^2 \quad (32)$$

It can be derived from the Lagrangian:

$$\mathcal{L} = \frac{1}{2} \frac{\dot{c}^2}{c^6} - \frac{1}{2} \frac{k}{c^2} + \frac{1}{6} \frac{\Lambda}{c^4} \quad (33)$$

whose associated Hamiltonian is the constant of motion:

$$\mathcal{H} = \frac{1}{2} \frac{\dot{c}^2}{c^6} + \frac{1}{2} \frac{k}{c^2} - \frac{1}{6} \frac{\Lambda}{c^4}, \quad \frac{d\mathcal{H}}{dt} = 0 \quad (34)$$

To each value of \mathcal{H} corresponds a sub-ensemble of models $\mathcal{S}(\Lambda, k; \mathcal{H})$, and if in particular $\mathcal{H} = 0$ then $\mathcal{S}(\Lambda, k; 0)$ is the ensemble of maximally symmetric space-time models. Therefore somehow the value of \mathcal{H} of a model of the ensemble $\mathcal{E}(\Lambda, k)$ measures its deviation with respect to the corresponding maximally symmetric model.

Using the data and the units mentioned at the end of the preceding section the value of \mathcal{H} for the basic universe described in this paper is 0.005.

From (4) and (34) it follows the eventually useful formula:

$$\rho = \frac{3}{4} \frac{c^4 \mathcal{H}}{\pi G} \quad (35)$$

Open question

Basic cosmology assumes that light from stars and galaxies propagates freely across the black-body radiation. But since it is known that while there is no direct photon-photon interaction there are indirect interactions through intermediate virtual particles [2], it could be ⁶ that the black body fluid has an index of refraction; and in this case light would not propagate along null geodesics of the Robertson-Walker model and cosmology would radically change from what now we believe it to be.

References

- [1] S. Weinberg, *Gravitation...* Chap. 13, John Wiley & Sons (1972)
- [2] D. d'Enterria and G. G. Silveira arXiv:1305.7142v2 [hep-ph]
- [3] Ll. Bel, arXiv:gr-qc/0306091 v1
- [4] Ll. Bel, arXiv:gr-qc/9905016 v1
- [5] Ö. Akarsu, T. Dereli, S. Kumar and L. Xu, arXiv:1305.5190v3 [gr-qc]

Figure 1 (c), **Figure 2** (ρ),
Figure 3 (Λ), **Figure 4** (\bar{k}).

⁶More on that in [4]

